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IX.—*On the General Theory of Duplex Telegraphy.* *By* Louis Schwendler.

(Continued from Vol. XLIII, Part II, 1874.)

In the two preceding investigations* I have given the solution of the *first problem* for the bridge method. This solution established the general result of the *double balance* being the best possible arrangement for the bridge method. In the present paper I shall endeavour to find the solution of the *first problem* for the *differential method*, which in practical importance ranges second to the bridge-method.

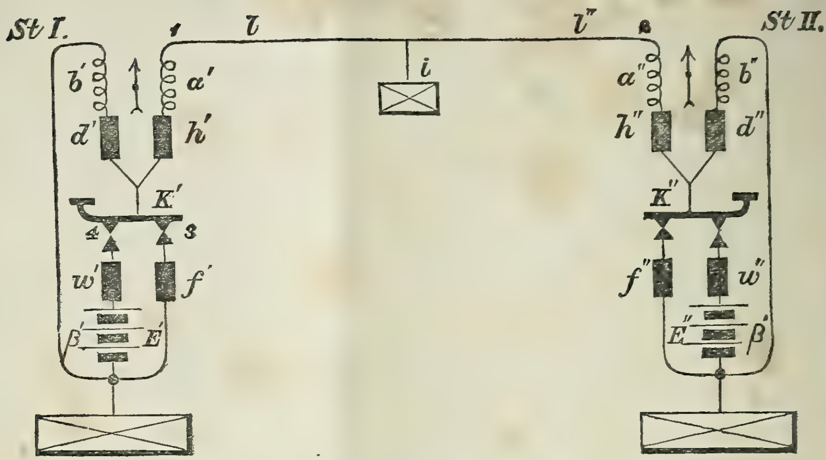
II. Differential method.†

This arrangement for duplex working is based on the well-known method, of comparing electrical resistances "differential method," and Fig. 2 gives the general diagram when this method is applied for duplex working.

* J. A. S. B., Vol. XLIII, Part II, 1874, pp. 1 and 218; Phil. Mag., Vol. 48, 1874, p. 117 and Vol. 49, 1875, p. 108; Journal Telegraphique, Vol. II, p. 580.

† The differential method was originally invented, as stated before, by Mr. Frischen, and Messrs. Siemens and Halske. A particular case of this method was patented by them in England in 1854.

Fig. 2.



Explanation of the Diagram.

E, electromotive force of the signalling battery.

β, internal resistance of the signalling battery.

k, a constant resistance key.

a and b, the coils of the receiving instrument. These coils, for any sent current, have opposite magnetic effects with respect to any given magnetic pole external to the coils; while for any received current, these coils add their effects with respect to that same magnetic pole. By a and b shall also be designated the resistances of the coils.

d, w, f, and h are certain resistances, the necessity of which will become clear hereafter.

i, the resistance of the resultant fault of the line, acting at a distance l' from Station I, and at a distance l'' from Station II, (both l' and l'' expressed in resistances, so that $l' + l'' = L$ equal the "real conduction resistance" of the line).

The other terms, *viz.* L', L'', ρ', ρ'', c', c'', &c., which will necessarily be of frequent occurrence also in this paper, will bear the same physical meaning here as before.

The practical inferiority of the *differential method*, when compared with the *bridge method*, it will be clear at once, is that specially constructed receiving instruments on the differential principle are required. That, therefore, the introduction of Duplex Telegraphy based on the differential method would at once involve also a total change of the receiving instruments hitherto used. This is clearly a serious disadvantage from an administrative and financial point of view. But besides this, without going into details, the differential method has also a very serious objection from a technical point of view. While in the bridge method the balance is obviously independent of the resistance of the receiving instrument, in the differential method the balance is clearly a function of the resistances of the two coils of which the receiving instrument consists, and as these two coils may alter their resistances independently, and not in proportion as indicated by the balance equation, a new element of disturbance is introduced, which the bridge method does not possess.

Besides this, differential instruments are necessarily mechanically more complicated than others, and require therefore superior workmanship, entailing greater expense to arrive at working efficiency.

General expressions for the two functions "D" *and* "S."

In order to obtain the two functions D and S, we have to develop the general expressions for p, P, and Q; say for Station I.

p' in our particular case is the force exerted by the two coils a' and b' on one and the same magnetic pole when Station I is sending and Station II is at rest. This force is clearly the difference of the two forces exerted by the coils a' and b'.

Thus we have

$$p' = A' m' - B' n'$$

where A' and B' are the currents which pass through the two coils a' and b' respectively, when Station I is sending and Station II is at rest, while m' and n' are the forces exerted by these coils when the unit current passes through them. At balance in Station I, $p' = o$

Further $$P' = \mathfrak{A}' m' + \mathfrak{B}' n'$$

where $\mathfrak{A}'$ and $\mathfrak{B}'$ are the currents which pass through the coils a' and b' respectively, when Station II is sending and Station I is at rest (single signals).

Further $$Q' = \gamma' m' + \delta' n'$$

where γ' and δ' are the currents which pass through a' and b' respectively when both stations are sending simultaneously (duplex signals).

To get the most general expressions for these three forces p, P, and Q, we have to fix the signs of the two terms of which they consist. This is best done by considering the forces m and n as absolute numbers, and determining the direction in which they act with respect to one and the same magnetic pole by the direction of the currents passing through the coils a and b.

To fix the signs of the currents, we shall call, arbitrarily, that current positive which passes through the coil a in the sending station, when the negative pole of the signalling battery is joined to earth.

Further, if we suppose at the outset, that the movement of the key k does not alter the complex resistance ρ of its own station, *i. e.*, the fulfilment of the key equation

$$w + \beta = f$$

a condition which is essential, it is clear that the currents $\mathfrak{V}'$ and $\mathfrak{g}'$ are the algebraical sums of the currents A', $\mathfrak{A}'$ and B', $\mathfrak{B}'$ respectively, whence it follows that

$$Q' = (A' + \mathfrak{A}')\, m' + (B' + \mathfrak{B}')\, n'$$

where the currents contain the signs.

Now, with respect to the manner of connecting up the two signalling batteries E' and E'', we have the following two different cases:

1*st.* The same pole of the signalling battery is connected to earth in each station, thus:

$$\begin{aligned} p' &= \pm A' m' \mp B' n' \\ P' &= \mp \mathfrak{A}' m' \mp \mathfrak{B}' n' \\ Q' &= (\pm A' \mp \mathfrak{A}')\, m' + (\mp B' \mp \mathfrak{B}')\, n' \end{aligned}$$

where the upper signs are to be used when the negative poles of the signalling batteries are connected to earth in both stations, and the lower signs when the positive poles of the signalling batteries are connected to earth in both stations.

2*nd.* Opposite poles of the signalling batteries are connected to earth in the two stations, thus:

$$\begin{aligned} p' &= \pm A' m' \mp B' n' \\ P' &= \pm \mathfrak{A}' m' \pm \mathfrak{B}' n' \\ Q' &= (\pm A' \pm \mathfrak{A}')\, m' + (\mp B' \pm \mathfrak{B}')\, n' \end{aligned}$$

where the upper signs are to be used when the negative pole in Station I and the positive pole in Station II are connected to earth, and the lower signs when the reverse is the case.

Subtracting in either of these two cases P' from Q', it will be seen that invariably

$$S' = Q' - P' = p'$$

or that, on account of having fulfilled the key equation $w + \beta = f$, the difference of force by which single and duplex signals are produced is equal in magnitude and sign to the force by which balance is disturbed. Further, that it is perfectly immaterial whether the same or opposite poles of the signalling batteries are put to earth. For reasons already explained I prefer to use the negative poles of the signalling batteries to earth in both stations, and this alternative we will suppose is adopted.

Thus we have:

$$\begin{aligned} p' &= A' m' - B' n' \\ P' &= -(\mathfrak{A}' m' + \mathfrak{B}' n') \\ Q' &= (A' - \mathfrak{A}')\, m' - (B' + \mathfrak{B}')\, n' \end{aligned}$$

If we now substitute for A', B', $\mathfrak{A}'$, $\mathfrak{B}'$ their values, we get :

$$p' = \frac{E'}{N'}\,\Delta'$$

$$P' = -\frac{E''\,(b'' + d'')}{N''}\,\mu'\,\lambda'$$

and $$Q' = -\frac{E''\,(b'' + d'')}{N''}\,\mu'\,\lambda' + \frac{E'}{N'}\,\Delta'$$

the sign of p' being contained in Δ', and
where

$$N' = f'\,(b' + d' + a' + h' + c') + (b' + d')\,(a' + h' + c')$$

$$N'' = f''\,(b'' + d'' + a'' + h'' + c'') + (b'' + d'')\,(a'' + h'' + c'')$$

$$\mu' = \frac{i}{i + l' + \rho'}$$

$$\Delta' = (b' + d')\,m' - (a' + h' + c')\,n'$$

$$\lambda' = m' + \frac{f'}{b' + d' + f'}\,n'$$

Thus the general expressions for the two functions D and S are :

$$\left.\begin{aligned} D' &= \frac{p'}{P'} = \frac{E'}{E''}\cdot\frac{N''}{N'\,(b'' + d'')}\cdot\frac{\Delta'}{\mu'\,\lambda'} \\ S' &= p' = \frac{E'}{N'}\,\Delta' \end{aligned}\right\}\text{for Station I.}$$

and $$\left.\begin{aligned} D'' &= \frac{p''}{P''} = \frac{E''}{E'}\cdot\frac{N'}{N''\,(b' + d')}\cdot\frac{\Delta''}{\mu''\,\lambda''} \\ S'' &= p'' = \frac{E''}{N''}\,\Delta' \end{aligned}\right\}\text{for Station II.}$$

Rigid fulfilment of the two functions $D = o$ and $S = o$.

D can only become zero, for finite resistances of the branches, if

$$p = S = o$$

i. e. if $$\Delta = o$$

Now, to keep $\Delta = o$ we may adopt two essentially different modes of re-adjustment, namely :—

Either leave the coils and their armatures stationary, and adjust balance by altering the resistances of the branches $(a + h)$ and $(b + d)$ separately or simultaneously, or leave the resistances of these branches constant, and move the coils or their armatures. These two cases are to be considered separately.

(*a.*) *Re-adjustment of balance by altering the resistances of the branches.*

As a and b are resistances which in the form of coils have to exert magnetic force, it is impracticable to suppose them variable. If they have been once selected, they must necessarily be kept constant, whence it follows that the re-adjustment of balance is restricted to a variation of the resistances h and d.

But as ρ is a function of h and d, to establish balance by altering one of them only, would invariably result in an alteration of ρ, and consequently *immediate* balance would become an impossibility.

Thus in order to readjust balance, and at the same time to keep ρ constant,* we must vary h and d simultaneously.

Now, it can be proved in exactly the same manner for the differential method as it was for the bridge, that in order to make the disturbance of balance for any given variation in the system as small as possible we must make ρ as large as possible, whence it follows from the form of ρ that

$$f = b + d$$

the "*regularity condition*" for the differential method.

But since

$$f = w + \beta$$

it follows that to re-establish balance by an alteration of the resistances h and d while a, b, β, and ρ keep constant, we have to vary *all* the four branches h, d, w and f simultaneously, in such a manner that their variations fulfil the following condition:

$$\delta f = \delta d = \delta w = -(2\ \delta h)$$

which is simple enough to allow of its practical application; but which nevertheless shows again the inferiority of the *differential method* as compared with the *double balance*, *i. e.*, in order to fulfil *immediate balance*, the *key equation*, and the *regularity condition* for the *differential* method, we have to make the four branches of the system simultaneously variable, while in the *double balance* the same effect can be obtained by having *one* branch only variable (the b branch).

It is worth while to mention here that there is a special case of obtaining *immediate balance* for the differential method by the adjustment in one branch, namely, when $f = o$, for then ρ would be independent of d, and therefore balance could be obtained by varying d without altering p.

However, on account of the key equation $f = w + \beta$, it would follow from $f = o$, that β must be zero also, which represents a physical impossibility inasmuch as the internal resistance of galvanic cells cannot be reduced

* $\rho = a + h + \dfrac{(b + d) f}{b + d + f}$

keep a, b and f constant and vary h and d, whence we should have:

$$\delta\rho = (b + d + f)\ (b + d + f + \delta d)\ \delta h + f^2\ \delta d = o$$

an equation, which it is always possible to fulfil for any variations of h and d if taken of opposite signs, although it may be difficult to achieve it practically by a simple motion, such as that of turning a handle. The absolute value of these variations depends of course on the variation of c which disturbs the balance, and in order to have accelerated balance we ought to decrease h and increase d when c increases, and *vice versá*.

to zero, not even approximately. Besides the E. M. F. requisite for duplex working being necessarily comparatively large, β will always be a quantity which cannot be neglected against the other resistances of the system, even if the single cells were of small resistance.

But supposing it were practicable to construct a battery of exceedingly low internal resistance, then, as $f = b + d$, it would be necessary to make $b = o$ and $d = o$ another physical impossibility, as b must consist of convolutions to produce magnetism, and d must be variable to produce balance.

This solution $f = b + d = w + \beta = o$, or even each of these three branches of an only exceedingly small resistance, must therefore be rejected.

(b.) *Adjustment of balance by moving the coils or armatures.*

This, it will be clear, is *the* solution for *immediate balance*, for such a mode of adjustment would involve no relation between the resistances of the three branches, leaving their determination free for other purposes. In order that the slightest movement of the two coils, or their armatures, may produce the required balance, it will be best to move both the coils or armatures simultaneously in the same direction. In fact to be able to produce balance, no matter how great the variation in the resistance of the line may become, it will be necessary to make the coils movable for the changes of sea sons, and the armatures for the daily changes.

It is clear that the differential method, when balance is adjusted by the movement of the coils or armatures, can alone be compared in efficiency with the *double balance*, and the superiority of the latter is most striking. While *immediate balance*, and the fulfilment of the other two essential conditions, can be obtained with *the double balance method* within any given range by a variation of the resistance in *one* single branch (b branch), this same result with the differential method can only be arrived at by either supposing four branches simultaneously variable, or by supposing the coils and armatures movable,—both pre-supposing complicated mechanical arrangements requiring delicate workmanship and being liable to get out of order.

Rapid approximation of the two functions D *and* S *towards zero.*

Supposing the fulfilment of the key equation as one of the most essential conditions, we know that

$p = S$ for each station invariably.

Now for Station I we have

$$p' = S' = E' \frac{\Delta'}{N'}$$

where

$$\Delta' = (b' + d')\, m' - (a' + h' + c')\, n'$$

$$N' = f'\,(b' + d' + a' + h' + c') + (b' + d')\,(a' + h' + c')$$

If we call c' that value of the measured circuit, which for any given values of the two branches $b' + d'$ and $a' + h'$ produces balance in Station

I, *i. e.* for which $\Delta' = o$, then if c' varies $\delta c'$, we have $\Delta' = n' \delta c'$, while N' becomes $N' + \delta N'$.

Thus we have

$$S' = E' \frac{n' \ \delta c'}{N' + \delta N'}$$

$$S' = \frac{E' n'}{f' + b' + d'} \cdot \frac{\delta c'}{a' + h' + \dfrac{f' (b' + d')}{f' + b' + d'} + c' + \delta c'}$$

but as $a' + h' + \dfrac{f (b' + d')}{f' + b' + d'} = \rho'$ the complex resistance in Station I, and as further $\delta c'$ can be neglected against c', we have finally:

$$S' = E' \frac{n'}{f' + b' + d'} \cdot \frac{\delta c'}{c' + \rho'}$$

Further n', the force exerted by the coil b' on a given magnetic pole when the unit current passes through the coil, can be expressed as follows:

$$n' = r' \sqrt{b'}*$$

where r' is a coefficient depending only on the dimensions and shape of the coil, on the manner of coiling the wire, and on the integral distance of the coil from the magnetic pole acted upon.

Thus we have

$$S' = E' \frac{r' \sqrt{b'}}{b' + f' + d'} \cdot \frac{\delta c'}{c' + \rho'} = E' . W' . \theta'$$

Now supposing the factor W' constant,† S' becomes smaller the smaller θ is.

In the second part it has been proved quite generally that θ decreases permanently with increasing $\rho' \ \rho''$, no matter to what special cause the variation of c' is due, whence again it follows that ρ should be a maximum.

From the form of ρ however we see that for any given sum $b + f + d$, ρ becomes largest if

$$f = b + d$$

which is "*the regularity condition*" of the differential method.

* This expression supposes that the thickness of the insulating covering of the wire can be neglected against the diameter of the wire, which is allowable. r' is a constant with respect to b'.

† That W' can be kept constant while θ' decreases and $\dfrac{f'}{b' + d'}$ varies, and $f' + b' + d'$ is constant, it will be clear is possible, for if $d' > o$ the variation of $b' + d'$ may be considered entirely due to a variation of d', equal and opposite in sign to the variation of f'. If $d' = o$ then we must consider r' variable with b' in order to keep W' constant while $\dfrac{f'}{b'}$ varies, which is admissible since the position of the coils has not been fixed as yet.

To have S' therefore for any variation as small as possible, we must make $f = b + d$. Substituting this value of f we get an expression for S' which shows that it has an absolute maximum for b but no minimum, from which we conclude that b should be made either very much smaller or very much larger than the value which corresponds to a maximum of S, but no fixed relation between b and d or a can be found.

In order to prove that $b + d = f$ is *the solution*, we must now show that it also makes D as small as possible.

But as $$D = \frac{S}{P}$$

we have only to show that the regularity condition $b + d = f$, makes P either as large as possible, or, which would be still better, a maximum.

Now

$$P' = A'' \mu' \lambda'$$

where A'' is the current which enters the line at point 2 (Fig. 2) when Station II is sending alone, while μ' is the factor which determines the loss through leakage of the line, and λ' is the factor to which the magnetic force, exerted by the current $A'' \mu'$ in Station I, is proportional.

μ' as well as λ' are functions of the resistances in Station I only* but not of those in Station II.

Now for constant values of μ' and λ' (*i. e.* leaving everything in Station I constant) P' becomes larger the larger A'' is:

$$A'' = E'' \frac{b'' + d''}{N''}$$

Substituting its value for N'', and dividing numerator and denominator by $b'' + d''$, we get

$$A'' = \frac{E''}{f'' + \frac{f''(a'' + h'')}{b'' + d''} + a'' + h'' + c''\left(1 + \frac{f''}{b'' + d''}\right)}$$

Supposing balance in Station II rigidly fulfilled, we have

$$(b'' + d'')\, m'' - (a'' + h'' + c'')\, n'' = o.$$

$$\therefore \quad c'' = (b'' + d'') \frac{m''}{n''} - (a'' + h'').$$

Substituting this value of c'' in the expression for A'' and reducing, we get

$$A'' = \frac{E'' r'' \sqrt{b''}}{f'' r'' \sqrt{b''} + q''(b'' + d'' + f'')\sqrt{a''}}$$

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* $\mu' = \frac{i}{i + l' + \rho'}$; $\lambda' = m' + \frac{f'}{f' + b' + d'} n'$

Dividing by q'', and putting $\frac{r''}{q''} = v''$ we have

$$A'' = E'' \frac{v'' \sqrt{b''}}{f'' v'' \sqrt{b''} + (b'' + d'' + f'' \sqrt{a''}}$$

This expression has a maximum* for

$$b'' = f'' + d''$$

which contradicts the regularity condition $f = b + d$ so long as d is different from zero.

Thus, in order to fulfil the regularity condition, and the maximum current, for the differential method simultaneously, we must put up

$$d = o$$

It has, however, been shewn that in order to have immediate balance, when adjusting balance by a variation in the resistances, we have to alter the resistances of the four branches $b + d$, $a + h, f$, and $w + \beta$ simultaneously according to a relation already given. Thus it is proved that adjustment of balance by an alteration of the resistances must be rejected, since, as pointed out before, a variation of the resistances of the coil b is impracticable.

We are obliged, therefore, to adjust balance by moving the coils or their armatures, and the further solution of the problem is only required, when this mode of adjustment is adopted.

Maximum magnetic moment.

It has now been proved that d is to be made zero, in order to be able to fulfil the conditions of *regularity* and *maximum current* simultaneously; and that therefore, to obtain immediate balance, readjustment of balance is to be effected by a movement of the two coils a and b or their armatures, and *not*, as has been generally proposed, by an alteration of the resistance in the branches $(a + h)$ and $(b + d)$.

Hence h appearing in the denominator of P only, and $h > o$ not being any more required for adjusting balance, the best value we can give to h is :—

$$h = o$$

which will make P, obviously largest.†

* In order to keep the balance in Station II rigid when b'' varies we must suppose v'' simultaneously variable with b''. This is perfectly justified, for v'' can be altered by an appropriate movement of the coils to keep up the balance in Station II without altering the outgoing current A''.

† The resistances d and h, without exerting magnetic force, were originally introduced in order to investigate the possibility of adjusting balance by an alteration of the resistances in the branches. But since it has been shown that this mode of adjustment is to be rejected it is of course clear that the dead resistances in these branches should be made zero when P will become largest.

Substituting therefore in the expression for P

$$h = d = o$$
$$f = w + \beta = b$$

we get

$$P' = \frac{E''}{2\,(a'' + c'') + b''}\,\mu'\,\lambda' \qquad \text{for Station I.}$$

and

$$P'' = \frac{E'}{2\,(a' + c') + b'}\,\mu''\,\lambda'' \qquad \text{,, Station II.}$$

These two expressions do not as yet contain the balance conditions.

The factors $\frac{\mu'}{2\,(a'' + c'') + b''}$ and $\frac{\mu'}{2\,(a' + c') + b'}$ are identical, namely :—

$$\frac{\mu'}{2\,(a'' + c'') + b''} = \frac{\mu''}{2\,(a' + c') + b'} = \frac{i}{Q}$$

Where $Q = i \left\{ 2\,(a' + a'' + l' + l'') + b' + b'' \right\} + \frac{b'\,b''}{2}$
$+ (a'' + l'')\,(a' + l' + b') + (a' + l')\,(a'' + l'' + b'')$
as can be easily calculated by sustituting for μ and c their known values.

In the second investigation it has been stated why P' and P'' cannot be made maxima separately, and that we could do nothing else but make their sum a maximum. In this case we have to do the same. Hence the question to be solved is reduced to the following :

$$P = P' + P'' = i.\ \frac{E''\,\lambda' + E'\,\lambda''}{Q}$$

is to be made a maximum with respect to the variables a, b, q and r, while they are linked together by two condition equations, namely :—

$$r'\,(a' + c') - q'\sqrt{a'\,b'} = o \text{ balance in Station I}$$

and $$r''\,(a'' + c'') - q''\sqrt{a''\,b''} = o \quad \text{,,} \qquad \text{,,} \quad \text{II}$$

This general problem can be solved in exactly the same way as it was in the second investigation. It is however not needed to do this again, since the general solution can be written down from inference, after having solved the special problem for a line which is perfect in insulation.

Suppose that $i = \infty$, or at least very large as compared with $l' + l' = L$, then obviously P' and P'' become identical without condition, namely :—

$$P' = P'' = P = \frac{E}{4}\,\frac{2\,q\sqrt{a} + r\sqrt{b}}{L + 2\,a + b}$$

while the two balance equations become also identical namely :—

$$2\,q\sqrt{a\,b} - r\,(4\,a + b + 2\,L) = o$$

If we substitute the value of r from the balance equation in the expression for P, we get

$$P = E\,q \cdot \frac{\sqrt{a}}{4\,a + 2\,L + b}$$

which has an absolute maximum with respect to a only, namely

$$a = \frac{L}{2} + \frac{b}{4}$$

Substituting this value of a in the last expression for P we get :

$$P = \frac{E\,q}{4} \cdot \frac{1}{\sqrt{2\,L + b}}$$

Whence it follows that P becomes largest for $b = o$, otherwise b remains indeterminate ; q on the other hand should be made as large as possible.

If we now put $v = \frac{r}{q}$, and develope its value from the balance equation, we get

$$v = \frac{r}{q} = \frac{1}{2}\sqrt{\frac{b}{2\,L + b}}$$

The solution of the 1st problem of the differential method, when the line is perfect in insulation, is therefore

$$h = d = o$$
$$f = b = w + \beta$$
$$a = \frac{L}{2} + \frac{b}{4}$$
$$v = \frac{1}{2}\sqrt{\frac{b}{2\,L + b}}$$

The absolute value of b is left indeterminate,* and we only know that the smaller it can be made the better.

But to fulfil this best condition $f = b = w + \beta = o$ represents a physical impossibility, since neither β, the internal resistance of constant galvanic cells, can be made zero, not even approximately, nor b, which must have convolutions in order to act magnetically.

The larger $f = b = w + \beta$ becomes, for practical reasons, the more the differential method, even under the best quantitative arrangements as given above, will become inefficient as compared with the double balance.

* Practically, however, it may be said, that b is given ; for generally β, the internal resistance of the signalling battery is determined by the nature and number of galvanic cells required for duplex working. We must only remember that b should be made somewhat larger than β, in order to have an adjustable resistance w in the battery branch, which may be used for compensating any variation of the battery resistance, that the equation $f = b = w + \beta$ may be permanently fulfilled.

Now by inference we get for a line with leakage, *i. e*, $i < \infty$

$$\left.\begin{aligned} a' &= \frac{L'}{2} + \frac{b'}{4} \\ a'' &= \frac{L''}{2} + \frac{b''}{4} \\ v' &= \frac{1}{2}\sqrt{\frac{b'}{2\,L' + b'}} \\ v'' &= \frac{1}{2}\sqrt{\frac{b''}{2\,L''+b''}} \end{aligned}\right\} \text{Approximately.}$$

The above values for a and v are somewhat too large, but in practical application they are quite correct enough.

The physical reason that this solution for the differential method gives an indeterminate result, is simply due to the fact that the force which produces the signals in the differential method is due to the combined magnetic actions of *two* separate coils through which unequal currents pass, instead of to *one* coil, as in the bridge method. On account of $b = f$, it follows that the current which passes through the b coil is only half of that passing through the a coil. Thus, in order to make the most of the arrived currents, b and f should be both equal to zero, or, in other words, placing *all* the convolutions in a and *none* in b must clearly give the greatest magnetic force. Obviously, however, such a solution could not fulfil the balance condition in the sending station.

The value of b should be chosen as small as practicable and its minimum value is β, the internal resistance of the signalling battery. How much larger b should be taken, depends on the absolute variation of β, *i. e.*, on the constancy of the resistance of the signalling battery. If the battery is very constant with respect to internal resistance, then b need be only very little larger than β, which determines the adjustable resistance w.

For instance minotto cells can be easily prepared with an internal resistance of 10 B. A. U. per single cell. Their minimum resistance, obtained by working, is never less than 5 B. A. U., and if the zincs are changed from time to time, their maximum resistance will scarcely ever be higher then 10 B. A. U.

Hence to make b about 50% larger than β will suffice, by which, if β is known, the greatest value of w is fixed.

The absolute value of β can be determined from the number of cells which have to be connected up successively, in order to work a given instrument through a given line, when the circuit Fig. 2 is adopted. This absolute value of β will therefore not only depend on the electrical state of the line and the nature of the cells, but also on the absolute sensitiveness of the differential instrument employed.

To make β therefore as small as possible, a sensitive construction of the differential instrument becomes requisite; further cells of high E. M. F. and low constant resistance are best adapted for forming the signalling battery. In order to get the widest limits in the variation of w it is clear that *that* β should be selected which is calculated from the maximum number of cells required to produce the signals with sufficient force. The greatest number of cells is obviously required when the line is at its lowest insulation, in India during the monsoon.

The value $v = \frac{r}{q}$ is what has been termed the mechanical arrangement of the differential instrument.*

If $b = w + \beta$ has been determined by fixing β, then v has its smallest value for L largest, which is the case when the line is perfect in insulation; when the coil a must be closest to the magnetic pole acted upon, and the coil b furthest away from it.

The highest value of v we obtain by substituting the lowest L, *i. e.* when the line is at its lowest insulation; when the coil b must be nearest to the magnetic point acted upon, and the coil a furthest away from it.

Hence the two limits of v being fixed by the known limits between which L varies, the extent of movement of the two coils is also fixed, and consequently, if q is chosen arbitrarily, the construction of the differential instrument is determined. But even q is not quite arbitrary, since we know the form, dimensions and resistance of the coils, which, for instance, in Siemens' polarized relays on any given line, have to produce the magnetism in single circuit to get the signals with engineering safety.

The solution of the 1st problem of the differential method is therefore:

1. Balance in each station must be obtained by a movement of the two acting coils or their armatures, either singly or better simultaneously in the same direction, and *not* by an alteration of the resistances in the branches.

2. If this mode of adjusting balance be adopted, then the solution is:

$$\mathrm{d} = \mathrm{h} = \mathrm{o}$$
$$\mathrm{f} = \mathrm{b} = \mathrm{w} + \beta$$
$$\mathrm{a} = \frac{\mathrm{L}}{2} + \frac{\mathrm{b}}{4}.$$
$$\mathrm{v} = \frac{\mathrm{r}}{\mathrm{q}} = \frac{1}{2}\sqrt{\frac{\mathrm{b}}{2\,\mathrm{L} + \mathrm{b}}}$$

It will now be clear that the given solution fulfils the following essential conditions:

* J. A. S. B., Vol. XLI, Pt. II, p. 148.
Phil. Mag., Vol. XLIV, p. 166.

(i). *Any variation of the resistance in the total system has the least possible disturbing effect on the receiving instrument.*

(ii). *Any disturbance of balance can be eliminated by an appropriate movement of the two acting coils or their armatures, without disturbing balance in the distant station.*

(iii). *Conditional maximum magnetic moment of the receiving instrument.*

(iv). *Conditional maximum current.*

Addendum I.

Here I wish to give some additional explanations and corrections with reference to the 1st and 2nd parts of this investigation.

In J. A. S. B., Vol. XLIII, 1874, Pt. II, p. 20, I have substituted

$$c' = L' + \rho''$$

without stating that this expression for c' is only approximately true. The correct expression for c' is clearly

$$c' = l' + \frac{i\,(l'' + \rho'')}{i + l'' + \rho''}$$

which approximates closely towards $L' + \rho''$ if $l'' + \rho''$ is sufficiently small as compared with i. This for any line in good electrical condition, will always be the case.

At page 9, in the foot note, for "*as nearly as possible equal*" read "*as nearly as possible proportional.*"

At page 20, $\dfrac{dG}{dg} = L\,(a^2 - g^2) + 2\,a\,g\,(d - g) = o$

should be $\dfrac{dG}{dg} = L\,(a^2 - g^2) + 2\,a\,(a\,d - g^2) = o$

At pages 19 and 224 after having shewn that

$$a + f = g + d$$

I conclude at once that on account of equation VI ($a\,d - g\,f = o$)

$$a = g = d = f \qquad \ldots \qquad \ldots \qquad \ldots \quad \text{VIII}$$

while mathematically it follows only that

$$a = g$$

and

$$d = f$$

These two equalities do certainly not contradict equation VIII but they do not necessitate it.

The additional reason why equation VIII should be chosen follows from the balance condition

$$a\,d - b\,c = o$$

$$\therefore \qquad b = \frac{a\,d}{c}$$

Therefore b becomes largest for any given c and any given $(a + d)$, if we put $a = d$.

But b largest is required for two separate reasons:

1. If the immediate balance is disturbed by an alteration of the resistance of one or more of the four branches, which may happen, especially by f, *i. e.*, β (battery resistance) varying, then ρ becomes at once a function of b, *i e.*, an increasing one with b. Thus in order to keep ρ as large as possible, and at the same time as constant as possible, b should be selected largest.

2. Further by making b as large as the circumstances will admit, we clearly have the largest sent and largest received currents, which will be clear without calculation. In fact later on, page 232, it has been shewn that $a = d$ is the condition for the maximum signalling current.

Addendum II.

Since the 3rd February, 1875, the main line from Bombay to Madras had been successfully worked duplicé by means of the "double balance method."

This line is worked direct, *i. e.*, without any translating instruments, and is 797 miles in length; it consists almost throughout of No. $5\frac{1}{2}$ wire B. W. G. (diameter $5\frac{1}{4}$ m. m.) and is supported chiefly on the Prussian insulator.

The section of this line from Bombay to Callian is exposed to the destructive influence of a tropical sea climate; between Callian and Poona the line passes over the Western Ghâts, the dense fogs during the cold weather and the heavy rains during the South-west monsoon on these hills seriously affect its insulation; from Poona to Sholapore and Bellary, the line runs inland and experiences a climate on the whole favourable for the maintenance of constant and high insulation; between Bellary and Madras, however, the line again comes under the influence of a most unfavourable climate, especially just before and during the continuation of the North-east monsoon, when the atmosphere at a high temperature, is saturated with moisture and salt, leaving conducting deposits on the surface of the insulators.

Consequently during the South-west monsoon the resultant fault is near Bombay, during the hot weather it shifts towards the middle of the line, and in November when the rains set in at Madras and the weather on the Bombay side is clearing up, the resultant fault is situated close to Madras.

By February next, duplex working will therefore have been submitted to a most severe test, applied as it will have been for a whole year to a long line the electrical condition of which is highly variable with respect to season and locality, and its practicability will doubtless again be clearly proved, as has already been the case on the Calcutta-Bombay line, 1600 miles, where under no more favourable climatic conditions, duplex has, for the past twelve months not only fulfilled but surpassed the expectations formed of it. No difficulties have been experienced, and it is believed never will be.

Strange as it may appear from a theoretical point of view, it will nevertheless be found in practice, that a line worked duplicé carries more than double the traffic of the same line worked singly ; for it represents two lines carried on different posts far distant from one another, instead of 2 parallel lines on the same posts, and consequently the highly injurious effects of voltaic induction are eliminated.

Further the receiving signallers, not being provided with keys, are unable to interfere with messages during their transmission.

Corrections and repetitions do not necessitate a stoppage of work, for they are obtained in the following manner: the receiving signaller marks with a cross, or underlines the words to be repeated, and places the message by the side of the sending signaller, who calls for the repetitions directly he has finished the message he is transmitting, and during this call the distant station may either send fresh messages or may also call for repetitions ; consequently single working need never be resorted to, and the simultaneous exchange of messages and corrections becomes continuous.

The Indian system of receiving (the sounder system which has now been universally recognised as the only right one hand for signalling) thus necessitates constant attention on the part of the receiving signallers, for any inattention on their part at once becomes known to the controlling officer.